Systems of Linear Inequalities in Two Variables

Key Definitions

- **<u>Half-Plane:</u>** The region on a side of a line in the xy-plane.

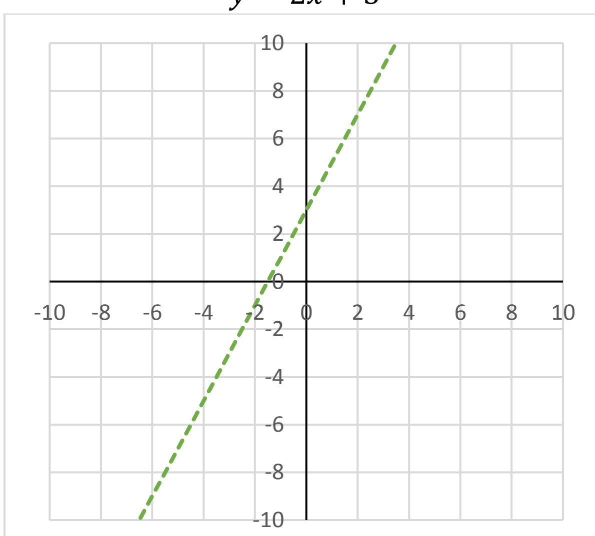
Graphing Linear Inequalities in Two Variables

- **<u>How to Graph Linear Inequalities in Two Variables:</u>**
  - **1.** Change the inequality sign to an equal sign, then plot the line.
    - If the inequality is $<$ or $>$, make the line **dashed**.
    - If the inequality is $\leq$ or $\geq$, make the line **solid.**
  - **2.** Test a point in one half plane created.
    - If it satisfies the inequality, the entire half-plane satisfies the inequality.
    - If it does not satisfy the inequality, the entire half-plane does not satisfy the inequality.
  - **3.** Test the other half-plane.
  - **4.** Shade in any half-planes that satisfy the inequality.
- **Example:** Graph the following inequality

$$y < 2x + 3$$

Step 1: Change the $<$ to $=$ and plot the line.

$$y = 2x + 3$$



***Notice that the line is <u>dashed</u> since the inequality is $<$.**

Step 2: Test a point in the half-plane to the left of the line. Let's use the point $(-1, 2)$ for this example.

$$y < 2x + 3$$

$$2 < 2(-1) + 3$$
$$2 < 1$$

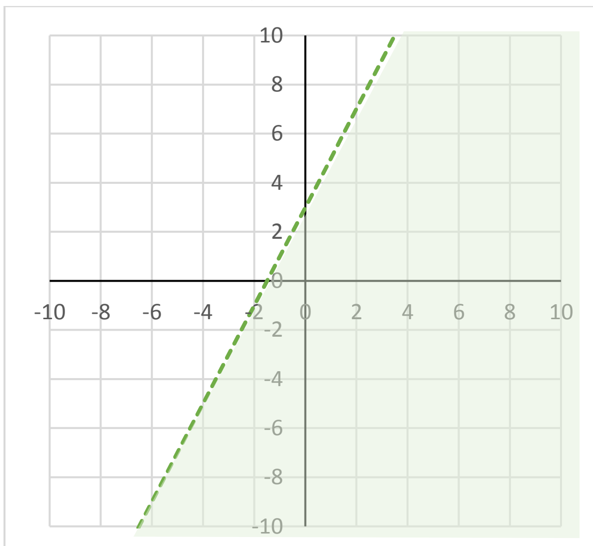
This point <u>does not</u> satisfy the inequality. Therefore, no point in the half-plane to the left of the line does not satisfy the inequality.

Step 3: Now test the other half-plane. For this example, let's use the point $(\mathbf{0}, \mathbf{0})$.

$$y < 2x + 3$$
$$0 < 2(0) + 3$$
$$0 < 3$$

This point <u>does</u> satisfy the inequality. Therefore, every point in the half-plane to the right of the line satisfies the inequality.

Step 4: Shade in the half-plane to the right of the line.



• Graphing Systems of Linear Inequalities in Two Variables

- To graph a system of linear inequalities in two variables, we want to find every possible x and y-value that satisfies both inequalities, similar to how we wanted every possible x and y-value that satisfies both equations when we were solving systems of equations.
- <u>**How to Solve a System of Linear Inequalities in Two Variables:**</u>
  - **1.** Using the technique of graphing inequalities above, graph both of the inequalities given.
  - **2.** Draw the completed graph shading *only* the overlapped shaded regions from the first step.
- **Example:** Graph the following system of inequalities.

$$y < 3x + 2$$

$$y \geq -\frac{1}{2}x + 1$$

Step 1: Change the inequality signs to equal signs and plot the lines accordingly.

$y < 3x + 2$	$y \geq -\frac{1}{2}x + 1$
$y = 3x + 2$	$y = -\frac{1}{2}x + 1$
Dotted Line	*Solid Line*

Testing Points:

$$y < 3x + 2$$

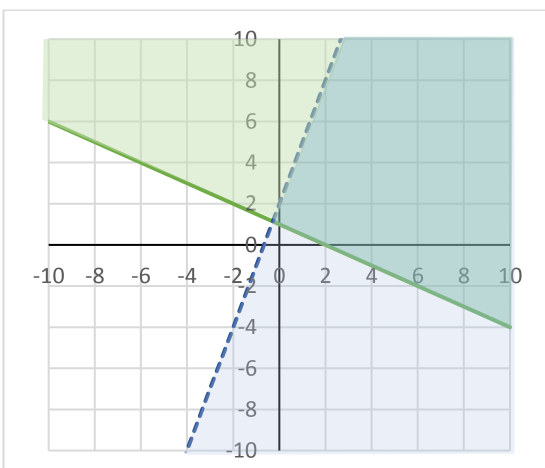
Test points $(-1,2)$ *and* $(0,0)$

$2 < 3(-1) + 2$	$0 < 3(0) + 2$
$2 < 1$	$0 < 2$
Does Not Satisfy	*Satisfies*
(left of line)	*(right of line)*

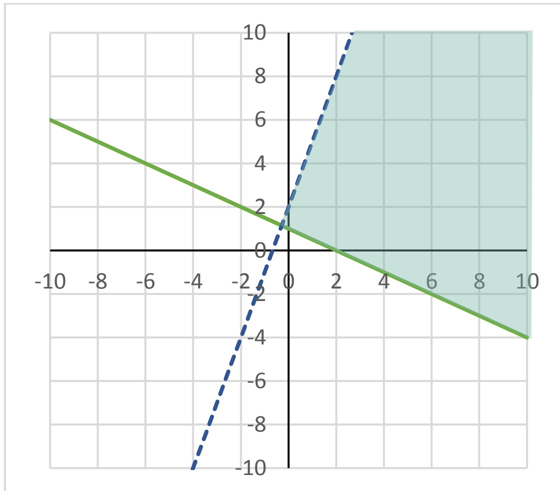
$$y \geq -\frac{1}{2}x + 1$$

Test points $(1,1)$ *and* $(1,-1)$

$1 < 3(1) + 2$	$1 < 3(-1) + 2$
$1 < 5$	$1 < -1$
Satisfies	*Does Not Satisfy*
(right of line)	*(left of line)*



Step 2: Create the complete graph only including the overlapping shading in the upper right region of the graph.



The Linear Programming Model

Key Definitions

- **Optimization:** The process of minimizing or maximizing a certain function.
- **Linear Programming:** The graphical approach of solving optimization problems.
- **Objective Function:** The function representing what we are trying to optimize
- **Constraints:** A system of linear inequalities that helps us find feasible solutions.
- **Feasible Solutions:** Any possible solution or outcome.
- **Vertex:** The points where the lines in the constraints meet and bound the region of feasible solutions

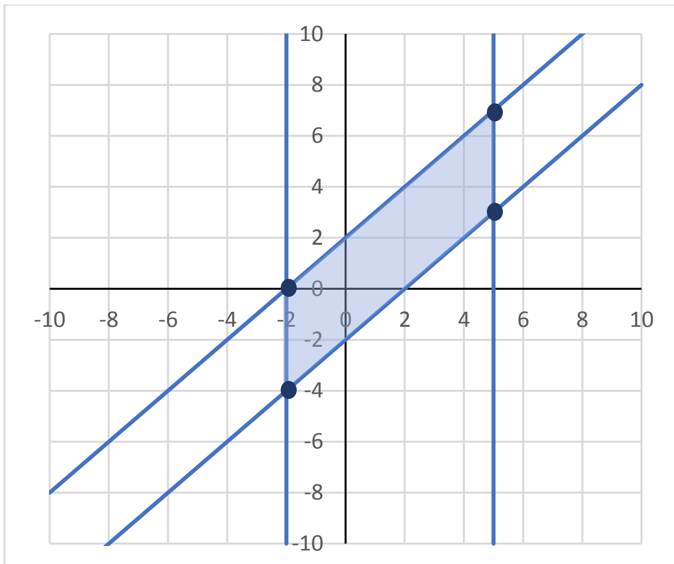
Optimizing Using the Linear Programming Model

- **How to Solve an Optimization Problem Using Linear Programming:**
  - **1.** Make sure that you are aware of your objective function and its constraints.
  - **2.** Graph the constraints (system of inequalities) so that we know what our feasible solutions are.
  - **3.** Identify the vertices and plug these values into the objective function.
  - **4.** Note that the smallest of these values you evaluates is the minimum and the largest value is the maximum.
- **Example:** Find the maximum and minimum value of the function $z = 4x - 2y + 1$ bounded by

$$x \leq 5, \qquad x \geq 2, \qquad y - x \leq 2, \qquad y - x \geq -2$$

Step 1: Since we are finding the minimum and maximum of $z = 4x - 2y + 1$, this makes it our objective function. In other words, we are trying to find the maximum and minimum z-values when x and y are constrained.
Since this function is bounded by $x \leq 5$, $x \geq 2$, $y - x \leq 2$, and $y - x \geq -2$, these are the constraints.

Step 2: Graph the system of inequalities (constraints).



Step 3: We see that vertices form at the points $(-2, 0), (-2, -4), (5, 7), (5, 3)$. Now, we will plug these values into the objective function.

$(-2,0)$: $\quad z = 4x - 2y + 1$

$z = 4(-2) - 2(0) + 1$

$z = -7$

$(-2,-4)$: $\quad z = 4x - 2y + 1$

$z = 4(-2) - 2(-4) + 1$

$z = 1$

$(5,7)$: $\quad z = 4x - 2y + 1$

$z = 4(5) - 2(7) + 1$

$z = 7$

$(5,3)$: $\quad z = 4x - 2y + 1$

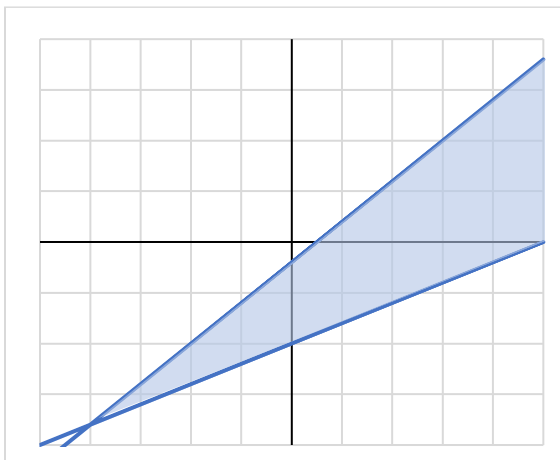
$$z = 4(5) - 2(3) + 1$$

$$z = 15$$

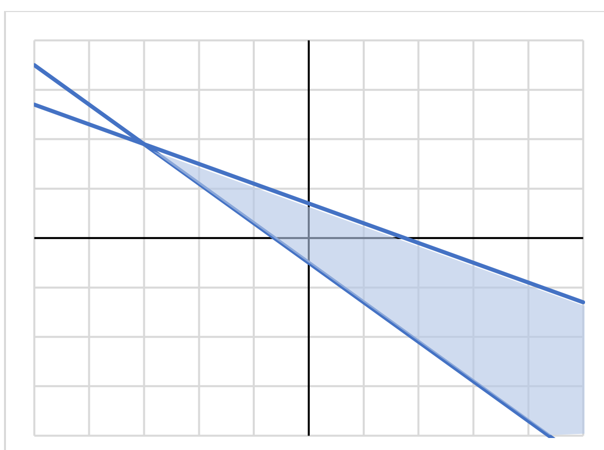
Step 4: Conclude that the maximum value is $z = 15$, which occurs at $x = 5$ and $y = 3$. Conclude that the minimum value is $z = -7$, which occurs at $x = -2$ and $y = 0$.

Optimizing Using the Linear Programming Model in Unbounded Regions

- If a given region is unbounded, that means that there is not an exact shape and your shaded region continues to go towards infinity or negative infinity.
  - If the region goes towards infinity, there is no maximum.
    - Visual Example:



  - If the region goes towards negative infinity, there is no minimum.
    - Visual Example:



  - If the region goes towards infinity and negative infinity, there is no maximum or minimum. In other words, it cannot be optimized. This only occurs if our constraints form parallel lines.

- Visual Example: