Measures of Dispersion

Key Definitions

- **<u>Range</u>:** The difference between the smallest and largest values in the data set.
- **<u>Population Variance</u>:** The population variance is the squared value of the population standard deviation.
- **<u>Population Standard Deviation</u>:** The population standard deviation shows how far away each value is from the mean on average. To use this standard deviation formula, your data set must be a population.
- **<u>Sample Variance</u>**: The sample variance is the squared value of the sample standard deviation.
- **<u>Sample Standard Deviation</u>:** The sample standard deviation shows how far away each value is from the mean on average. To use this standard deviation formula, your data set must be a sample.
- **<u>Class Midpoint</u>:** The sum of two consecutive lower class limits that are then divide by 2.
- **<u>Approximate Standard Deviation of a Variable from a Frequency Distribution</u>:** The standard deviation for a frequency distribution shows approximately how far each value is away from the mean based on an average. This could be used for either a sample data set or a population data set. It is important to remember that the sum of the frequencies is the same as a sample or population size, and that we have to multiply each squaring (like the variance for both population and sample) by the frequency since that is the amount of times the value has appeared in a data set.

Population Variance and Standard Deviation

- ***How to Find the Population Variance:*** To find the population, we take each value from our data set, subtract the population mean from it, and square the result of the subtraction. Afterwards, we add together all of the results of the squaring. We take the sum just calculated and divide the population size from it. This will give us the population variance. The following is the formula:

$$\sigma^2 = \frac{(x_i - \mu)^2}{N}$$

- ***How to Find the Population Standard Deviation:*** After we have found the population variance, it is easy to find the population standard deviation. We take the square root of the population variance, and that gives us the population standard deviation. The formula is the following:

$$\sigma = \sqrt{\sigma^2}$$

- ***Example of Population Variance and Standard Deviation:***
  You are given a population data set of the ages of kids who are taking swim lessons at a swimming facility as well as the population data set's mean. The mean is $\mu = 8$ and the data is the following:

  10, 5, 9, 7, 11, 6

  To find the variance and standard deviation of the data set, we will do each step broken down in the table down below:

Values (x)	$(x - \mu)$	$(x - \mu)^2$
10	$(10 - 8) = 2$	$(2)^2 = 4$
5	$(5 - 8) = -3$	$(-3)^2 = 9$
9	$(9 - 8) = 1$	$(1)^2 = 1$
7	$(7 - 8) = -1$	$(-1)^2 = 1$
11	$(11 - 8) = 3$	$(3)^2 = 9$
6	$(6 - 8) = -2$	$(-2)^2 = 4$
Total:		28

$$\sigma^2 = \frac{(x - \mu)^2}{N} = \frac{28}{6} \approx 4.67$$

Now that we have found the variance, we can find the standard deviation very easily. We are going to take the square root of the variance, and that will give us the standard deviation.

$$\sigma = \sqrt{\sigma^2} = \sqrt{4.67} \approx 2.16$$

* There is an easier way to compute the population variance and standard deviation in Excel. If you would like to learn how to use Excel to compute the population variance and standard deviation, we have an Excel spreadsheet with instructions on our website.

Sample Variance and Standard Deviation

- ***How to Find the Sample Variance:*** To find the population, we take each value from our data set, subtract the sample mean from it, and square the result of the subtraction. Afterwards, we add together all of the results of the squaring. We take the sum just calculated and divide the sample size from it. This will give us the sample variance. The following is the formula:

$$s^2 = \frac{(x_i - \bar{x})^2}{n - 1}$$

- ***How to Find the Sample Standard Deviation:*** After we have found the sample variance, it is easy to find the sample standard deviation. We take the square root of the sample variance, and that gives us the sample standard deviation. The formula is the following:

$$s = \sqrt{s^2}$$

- ***Example of Sample Variance and Standard Deviation:***
  You are given a sample data set of test scores 10 students received on their exams as well as the sample data set's mean. The mean is $\bar{x} = 82.3$ and the data is the following:

  90, 98, 94, 75, 78, 83, 75, 72, 73, 85

  To find the variance and standard deviation of the data set, follow the step broken down in the table down below:

Values (x)	$(x - \bar{x})$	$(x - \bar{x})^2$
90	$(90 - 82.3) = 7.7$	$(7.7)^2 = 59.29$
98	$(98 - 82.3) = 15.7$	$(15.7)^2 = 246.49$
94	$(94 - 82.3) = 11.7$	$(11.7)^2 = 136.89$
75	$(75 - 82.3) = -7.3$	$(-7.3)^2 = 53.29$
78	$(78 - 82.3) = -4.3$	$(-4.3)^2 = 18.49$
83	$(83 - 82.3) = 0.7$	$(0.7)^2 = 0.49$

75	$(75 - 82.3) = -7.3$	$(-7.3)^2 = 53.29$
72	$(72 - 82.3) = -10.3$	$(-10.3)^2 = 106.09$
73	$(73 - 82.3) = -9.3$	$(-9.3)^2 = 86.49$
85	$(85 - 82.3) = 2.7$	$(2.7)^2 = 7.29$
Total:		768.1

$$s^2 = \frac{(x - \bar{x})^2}{n - 1} = \frac{768.1}{10 - 1} \approx 85.34$$

Now that we have found the variance, we can find the standard deviation very easily. We are going to take the square root of the variance, and that will give us the standard deviation.

$$s = \sqrt{s^2} = \sqrt{85.34} \approx 9.24$$

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Approximate Standard Deviation of a Variable from a Frequency Distribution

- ***How to Find the Class Midpoint:*** To find the class midpoint, you take the two consecutive lower class limits (the first lower class limit is the one for the class you are finding the midpoint and the next lower class limit is the next class that is after the class you are finding the midpoint for), add them together, and then divide the sum by 2. The midpoint of a frequency distribution is represented by *x*.
- ***How to Find the Standard Deviation of a Frequency Distribution:*** To find the standard deviation of a frequency distribution, you take each value from the data set, subtract the mean from each value, square each subtraction, and then multiply the squaring by the frequency for the value. Afterward, we add together the multiplication of the frequency and squaring. Depending on whether the data set from the frequency distribution is a sample or population set, you follow different steps. If you have a population data set, you divide the sum by the total frequencies, and then take the square root of the division. If you have a sample data set, you divide the sum by the total frequencies minus 1, and then take the square root of the division. The following is the two formulas:

  Sample Frequency Distribution

  $$s = \sqrt{\frac{\sum (x_i - \bar{x})^2 f_i}{\sum f_i - 1}}$$

  Population Frequency Distribution

  $$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2 f_i}{\sum f_i}}$$

- ***Example of Standard Deviation of a Frequency Distribution:***
  You are given the information about a fast food chain that took a sample of how much their customers spend at their store. The following is their findings presented in a frequency distribution and the mean of the frequency:

  $$\bar{x} = 6.1$$

Classes	Frequency
\$1.00 - \$3.99	7
\$4.00 – \$6.99	5
\$7.00 - \$9.99	5
\$10.00 - \$12.99	3

We use a chart to break down the steps we need to take. We find the midpoints, and then complete each of the steps.

Classes	Frequency	Midpoint (x)	$(x - \bar{x})$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
\$1.00 - \$3.99	7	$\frac{4-1}{2} = 2.5$	(2.5 – 6.1) = -3.6	$(-3.6)^2 = 12.96$	$(12.96)(7) = 90.72$
\$4.00 - \$6.99	5	$\frac{7-4}{2} = 5.5$	(5.5 – 6.1) = -0.6	$(-0.6)^2 = 0.36$	$(0.36)(5) = 1.8$
\$7.00 - \$9.99	5	$\frac{10-7}{2} = 8.5$	(8.5 – 6.1) = 2.4	$(2.4)^2 = 5.76$	$(5.76)(5) = 28.8$
\$10.00 - \$12.99	3	$\frac{13-10}{2} = 11.5$	(11.5 – 6.1) = 5.4	$(5.4)^2 = 29.16$	$(29.16)(3) = 87.48$
Totals:	20				208.8

Now, we plug in the information into the equation:

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2 f_i}{\sum f_i - 1}} = \sqrt{\frac{208.8}{20 - 1}} = 3.32$$

Symbol Guide

Chapter Title Symbols		
Term	Symbol	Use
Range	R	Identify the range
Population Variance	σ^2	Identify the population variance
Population Standard Deviation	σ	Identify the population standard deviation
Sample Variance	s^2	Identify the sample variance
Sample Standard Deviation	s	Identify the sample standard deviation
Frequency	f	Identify the frequency