Hypothesis Testing

Key Definitions

- **Hypothesis:** A statement about a characteristic of one or more populations.
- **Hypothesis Testing**: The procedure used to test the statement made about a characteristic regarding one or more populations.
- **Null Hypothesis**: The statement that you are testing. This statement is assumed to be true as it is the original statement made. It is represented by a H_0.
- **Alternative Hypothesis**: The new statement that we are trying to prove is correct instead of the null hypothesis. It is represented by a H_1.
- **Parameter:** The statistic that you are testing the truth value of. This is something like the mean or standard deviation.
- **Level of Significance**: The probability of making a type I error (see below).

Setting Up the Hypothesis

- *Two-Tailed Test:* You first must determine the parameter. After determining the parameter, you must identify the original observation and the new statement you are trying to prove. Then, you put it in the following form:

$$H_0: parameter = value$$
$$H_1: parameter \neq value$$

- *Example of a Two-Tailed Test Hypothesis:*
  This price for the *Fundamentals of Statistics* 5th *edition* is sold for the mean price of $152.00. Sally believes that the mean price of the textbook is not $152.00

$$H_0: \mu = 152.00$$
$$H_1: \mu \neq 152.00$$

- *Right-Tailed Test:* You first must determine the parameter. After determining the parameter, you must identify the original observation and the new statement you are trying to prove. Then, you put it in the following form:

$$H_0: parameter = value$$
$$H_1: parameter > value$$

- *Example of a Right-Tailed Test Hypothesis:*
  The mean monthly money spent for a customer at a fast food company is $25.15. A researcher believes that the mean is more than $25.15.

$$H_0: \mu = 25.15$$
$$H_1: \mu > 25.15$$

- *Left-Tailed Test:* You first must determine the parameter. After determining the parameter, you must identify the original observation and the new statement you are trying to prove. Then, you put it in the following form:

$$H_0: parameter = value$$
$$H_1: parameter < value$$

- *Example of a Left-Tailed Test Hypothesis:*

The standard deviation of children in swimming lesson is 2 kids. The owner of the facility believes it is less than that.

$$H_0: \sigma = 2$$
$$H_1: \sigma < 2$$

Types of Error

- **_Type I Error:_** This error occurs when our null hypothesis was the true outcome, but we determined the alternative hypothesis was true.
- **_Type II Error:_** This error occurs when our alternative hypothesis was the true outcome, but we determined the null hypothesis was true.
- **_Diagram for Errors:_**

		Reality	
		$H_o\ is\ True$	$H_1\ is\ True$
Conclusion	$Do\ Not\ Reject\ H_0$	Correct Conclusion	Type II Error
	$Reject\ the\ H_0$	Type I Error	Correct Conclusion

Level of Significance

- **_Finding Critical Values for Left-Tailed:_** When we are given a level of significance, we are able to find the critical value that goes along with it. First, we use our α (alpha) as our area/probability in the standard normal distribution chart. Once we find the value in the chart, we align it with the proper z-score. This gives us the critical value.
- **_Example of Finding a Critical Value for a Left-Tailed:_**
  The standard deviation of children in swimming lesson is 2 kids. The owner of the facility believes it is less than that. The level of significance is $\alpha = 0.05$.
  We look into the Standard Normal Distribution chart in the back of the textbook. Inside of the chart, we look for the area that is closest to 0.05. We see that the z-scores -1.64 and -1.65 has area where our level of significance is perfectly in-between. Since it is perfectly in-between, we go with -1.64 but add a 0.005 to the decimal since it is perfectly centered. So our critical value will be
  $$z_{0.05} = -1.645.$$
- **_Finding Critical Values for Right-Tailed:_** When we are given a level of significance, we are able to find the critical value that goes along with it. First, we take the α and subtract it from 1. Then, we use our $1 - \alpha$ as our area/probability in the standard normal distribution chart. Once we find it in the chart, we align it with the proper z-score. This gives us the critical value.
- **_Example of Finding a Critical Value for a Right-Tailed:_**

The mean monthly money spent for a customer at a fast food company is $25.15. A researcher believes that the mean is more than $25.15. The level of significance is $\alpha = 0.01$.

Since we are dealing with a right-tailed test, our level of significance we need to take our level of significance and subtract it from 1 to find our new area that properly aligns with a right-hand test. When we do this, we get $1 - \alpha = 1 - 0.01 = 0.99$. Next, we look at our standard normal distribution chart to find our critical value. When we do this, we find our critical value is

$$z_{0.99} = 2.33$$

- **_Finding Critical Values for Two-Tailed:_** When we are given a level of significance, we are able to find the critical value that goes along with it. First, we take the α and divide it by 2. Then, we use the division as our area/probability in the standard normal distribution chart. Once we find it in the chart, we align it with the proper z-score. This gives us one critical value. To find the second critical value, we simply take the z-score and make it negative.

- **_Example of a Finding Critical Value for a Two-Tailed:_**

This price for the _Fundamentals of Statistics_ 5th _edition_ is sold for the mean price of $152.00. Sally believes that the mean price of the textbook is not $152.00. The level of significance is $\alpha = 0.10$. Since we are dealing with a two-tailed test, we must take our level of significance and divide it by 2. When we do this, we get $\frac{0.10}{2} = .05$. Next, we look at the standard normal distribution chart to find the critical value of the area. We get that it is

$$z_{0.05} = -1.645$$

Since we have a two tailed test, which is not the only critical value. The other critical value is just has the opposite sign of the one you already found. Therefore, our answer is:

-1.645, 1.645

Symbol Guide

Chapter Title Symbols		
Term	Symbol	Use
Null Hypothesis	H_0	To identify the null hypothesis
Alternative Hypothesis	H_1	To identify the alternative hypothesis
Level of Significance	α	To identify critical values