Discrete Probability Distribution

Key Definitions

- **Discrete Random Variable:** Has a countable number of values. This means that each data point is distinct and separate.
- **Continuous Random Variable:** Has an infinite number of values. This means that each data point runs into the next. They are not individually countable.
- **Probability Distribution of a Discrete Random Variable:** Has the possible values of the random variable and the probabilities that correspond with each random variable.
- **Binomial Probability Distribution:** A discrete probability distribution that describes the probabilities of the experiments in which there are two outcomes that are independent of one another.

Rules of a Discrete Probability Distribution

- ***Rules of a Discrete Probability Distribution:*** There are two different rules that all discrete probability distribution:
  - The sum of all probabilities must equal 1. This is formally written as:

$$\sum P(x) = 1$$

  - All probabilities must be between 0 and 1. This is formally written as:

$$0 \le P(x) \le 1$$

- ***Example for Using the Rules of a Discrete Probability Distribution:***
Determine if the following is a discrete probability distribution:

x	$P(x)$
1	0.15
2	0.24
3	0.36
4	0.40
5	-0.15

We first check to see that when we add up all the probabilities, they equal 1.

$$\sum P(x) = 0.15 + 0.24 + 0.36 + 0.40 - 0.15 = 1$$

The next thing to notice is that $P(5) = -0.15$. This is a negative probability and is against the rules of a discrete probability distribution. This means this example is not a probability distribution.

The Mean and Expected Value of a Discrete Random Variable

- ***How to Find the Mean/Expected Value:*** To find the Mean (also known as the Expected Value) of a discrete random variable, we take each x-value and multiply it by its probability. We then add together all of the multiplications. This is represented by the following formula:

$$\mu_X = \sum [\, x \cdot P(x)]$$

- ***Example of Finding the Mean/Expected Value:***
Determine the mean of the following discrete probability distribution:

x	$P(x)$
1	0.04
2	0.24
3	0.36
4	0.30
5	0.06

We need to multiply each x by its P(x):

x	$P(x)$	$x \cdot P(x)$
1	0.04	$1(0.04) = 0.04$
2	0.24	$2(0.24) = 0.48$
3	0.36	$3(0.36) = 1.08$
4	0.30	$4(0.30) = 1.20$
5	0.06	$5(0.06) = 0.30$

We now add up all of the multiplication done to get the final answer:

$$\mu_X = 0.04 + 0.48 + 1.08 + 1.20 + 0.30 = 3.1$$

The Standard Deviation of a Discrete Random Variable

- ***How to Find the Standard Deviation:*** The method used to find the standard deviation is very similar to the method used when finding the standard deviation of a frequency distribution. How we find the standard deviation of a discrete random variable is by subtracting each x value by the mean. After we subtract, we square the subtraction. After squaring the subtraction, we multiply it by corresponding probability for the x-value. After we have done this for each x-value, we add up the multiplication, and take the square root of the sum. This is represented by the following formula:

$$\sigma_X = \sqrt{\sum \, [(x - \mu_X) \cdot P(x)]}$$

- ***Example of Finding the Standard Deviation:***
Using the data from the example of finding the mean of a discrete random variable, we will create a table to continue working on our information. We know the mean is 3.1.

x	$P(x)$	$(x - \mu_X)$	$(x - \mu_X)^2$	$(x - \mu_X)^2 \cdot P(x)$
1	0.04	$(1 - 3.1) = -2.1$	$(-2.1)^2 = 4.41$	$(4.41)(0.04) = 0.1764$
2	0.24	$(2 - 3.1) = -1.1$	$(-1.1)^2 = 1.21$	$(1.21)(0.24) = 0.2904$
3	0.36	$(3 - 3.1) = -0.1$	$(-0.1)^2 = 0.01$	$(0.01)(0.36) = 0.0036$
4	0.30	$(4 - 3.1) = 0.9$	$(0.9)^2 = 0.81$	$(0.81)(0.3) = 0.243$
5	0.06	$(5 - 3.1) = 1.9$	$(1.9)^2 = 3.61$	$(0.06)(3.61) = 0.2166$
				0.93

Now, we plug our sum into the equation:

$$\sigma_X = \sqrt{\Sigma\,[(x - \mu_X) \cdot P(x)]} = \sqrt{0.93} = 0.96$$

Determining if the Probability Experiment is a Binomial Experiment

- ***Criteria for a Binomial Probability:*** If the experiment has all of the following, it is a binomial probability experiment:
  - The experiment is performed a fixed number of times. Each time the experiment is run, it is called a trial.
  - Each trial is independent of the others (the outcome of one trial does not affect the outcomes of the other trials).
  - There are two mutually exclusive outcome: success and failure.
  - The probability of success is the same for each experiment.
- ***Notation Used for the Binomial Probability Experiment:***
  - Number of independent trials: n
  - Probability of success for each trial: p
  - Probability of failure for each trial: $1 - p$
  - The number of successes in the n independent trials: x
  - The number of successes must be less than or equal to amount of trials: $0 \leq x \leq n$
- ***Example of Determining if it is a Binomial Probability:***
  An experiment is conducted in which 93% of students who go to the Math Tutoring Lab regularly pass their math course. Suppose a random sample size of 8 is obtained, and the number that passed is recorded. Is this a binomial probability?
  - The amount of trials: $n = 8$
  - Are the trials independent: Yes because each student passing or failing is independent of one another.
  - Are there two solutions: Yes. It is either passing or failing.
  - The probability: the probability is the same every time with $93\% = 0.93$ passing, and $1 - .93 = 0.07$ being the probability of failing.

  This is a binomial probability since it meets all of the criteria.

Computing the Probability of a Binomial Probability Experiment

- ***The Binomial Distribution Function:*** This function is the formula we use to find the probability of the number of successes based on the given independent trials, probability of success, probability of failure, and the number of successes. To find the probability, we plug into the following function:
  $$P(x) = C(n, x) \cdot p^x \cdot (1 - p)^{n-x}$$
  We find this by computing the Combination of n trials choosing x number of successes. We then find the probability of successes raised to the number of successes power. Then, we find the probability of failure brought to the trials minus number of successes power. Then, we multiply each one of the answers together to get the probability.
- ***How to Find the Probability of a Single Success Using the Binomial Distribution Function:*** When x equals only one number, we simply plug into the Binomial Distribution Function to find the probability of that success out of multiple successes.

- ***Example of Finding the Probability of a Single Success Using the Binomial Distribution Function:*** An experiment is conducted in which 93% of students who go to the Math Tutoring Lab regularly pass their math course. Suppose a random sample size of 8 is obtained, and the number that passed is recorded. What is the probability of getting 2 successes?
First, we identify the information that we know:
$$n = 8\ ,\ x = 2, p = 0.93,\ and\ (1 - p) = 0.07$$
Now, we plug into the formula of the Binomial Distribution Function:
$$P(x) = C(n, x) \cdot p^x \cdot (1 - p)^{n-x}$$
$$P(2) = C(8,2) \cdot (0.93)^2 \cdot (0.07)^6$$
$$P(2) = 28(0.8649)(0.000000117649) = 0.0000028$$
So, the probability of only 2 people passing their class is 0.0000028 when they come to the math tutoring lab.
- ***How to Find the Probability of Multiple Successes Using the Binomial Distribution Function:*** When x equals more than one number, we plug into the Binomial Distribution Function to find the probability of each success out of multiple successes for as many number of successes we have. After we have found each probability, we add them together to find the probability for multiple successes.
- ***Example of Finding the Probability of Multiple Successes Using the Binomial Distribution Function:*** An experiment is conducted in which 93% of students who go to the Math Tutoring Lab regularly pass there course. Suppose a random sample size of 8 is obtained, and the number that passed is recorded. What is the probability of getting less than 3 successes?
Since we are getting less than 3 successes, this means that we will look into the probability of them getting 0, 1, or 2 successes. So, now we must identify the rest of the information:
$$n = 8\ ,\ x = 0, 1, 2, p = 0.93,\ and\ (1 - p) = 0.07$$
Now, we plug into the formula 3 different times for each individual x. We will round all of final probability answers to the 7th decimal place. First, we start with $x = 0$:
$$P(x) = C(n, x) \cdot p^x \cdot (1 - p)^{n-x}$$
$$P(0) = C(8,0) \cdot (0.93)^0 \cdot (0.07)^8$$
$$P(0) = 1(1)(0.00000000057648) = 0.0000000$$
Next, we will plug in for $x = 1$:
$$P(x) = C(n, x) \cdot p^x \cdot (1 - p)^{n-x}$$
$$P(1) = C(8,1) \cdot (0.93)^1 \cdot (0.07)^7$$
$$P(1) = 8(0.93)(0.00000000823543) = 0.0000000$$
Then, we will plug in for $x = 2$. Since we already know the answer from the previous example, we will just put that down below:
$$P(2) = 0.0000028$$
Then, we add up all of the individual probabilities to find the probability of multiple successes:
$$P(0) + P(1) + P(2) = 0.0000000 + 0.0000000 + 0.0000028 = 0.0000028$$
The probability of less than 3 students passing their classes while using the Math Tutoring Lab is 0.0000028.

To see an easier way to find the probability of a Binomial Probability Distribution, look at our Excel handout

The Mean and Standard Deviation of a Binomial Probability Experiment

- ***How to Find the Mean of a Binomial Probability Experiment:*** To find the mean of the binomial probability experiment, we use a special formula. This formula is the following:

  $$\mu_x = np$$

  To find the mean, we simply take the number of trials and multiply it by the probability of success.
- ***How to Find the Standard Deviation of a Binomial Probability Experiment:*** To find the standard deviation of the binomial probability experiment, we use the following formula:

  $$\sigma_x = \sqrt{np(1-p)}$$

  To find the standard deviation, we take the mean and multiply it by the probability of failure. Then, we take the answer of the multiplication and take the square root of it.
- ***Example of How to Find the Mean and Standard Deviation of a Binomial Probability Experiment:*** An experiment is conducted in which 93% of students who go to the Math Tutoring Lab regularly pass their course. Suppose a random sample size of 8 is obtained, and the number that passed is recorded. What is the mean and standard deviation of this binomial probability experiment?
  First, we want to identify the information we know:

  $$n = 8,\ p = 0.93,\ and\ (1-p) = 0.07$$

  Now, we want to find the mean first. To do this, we plug into the formula above:

  $$\mu_x = np$$
  $$\mu_x = 8(0.93) = 7.44$$

  This shows that the mean of the binomial probability experiment is 7.44.
  Next, we find the standard deviation. We do this by plugging into the formula and rounding our answer to 2 decimal places:

  $$\sigma_x = \sqrt{np(1-p)}$$
  $$\sigma_x = \sqrt{7.44(0.07)} = \sqrt{0.5208} = 0.72$$

  The standard deviation of the binomial probability experiment is 0.72

Symbol Guide

Chapter Title Symbols		
Term	Symbol	Use
Mean/Expected Value of a Discrete Random Variable	μ_X	To identify the mean/expected value of a discrete random variable
Standard Deviation of a Discrete Random Variable	σ_X	To identify the standard deviation of a discrete random variable
Number of Trials	n	To identify the number of trials
Number of Successes	x	To identify the number of successes
Probability of Success	p	To identify the probability of success
Probability of Failure	$(1-p)$	To identify the probability of failure
Combination	$C(n,x)$	To identify the number of ways to choose x out of n

Mean of a Binomial Probability Experiment	μ_x	To identify the mean of the binomial probability experiment
Standard Deviation of a Binomial Probability Experiment	σ_x	To identify the standard deviation of the binomial probability experiment