Geometry

Key Definitions

- **<u>Polygon</u>:** A polygon is a two-dimensional, closed figure. Many polygons are named for the number of sides that they have.
- **<u>Perimeter</u>:** The perimeter of any polygon is the total distance around the figure, represented by P. This is found by adding each side of the figure.
- **<u>Area</u>:** The area of a polygon is the total space inside of the figure, measured in square units and denoted by A. Area is measured using base and height, represented by b and h, respectively. In this case, h is the actual vertical distance between horizontal bases.
- **<u>Right Triangle</u>:** A right triangle is a triangle with a right angle ($90°$). The longest side (opposite the right angle) is called the hypotenuse and the other two sides are called the legs.
- **<u>Circle</u>:** A circle is the set of all points the same distance away from a determined center point.
- **<u>Radius</u>:** The radius of a circle is the linear distance from the center of a circle to any point on the edge of the circle, denoted r.
- **<u>Diameter</u>:** The diameter of a circle is the straight-line distance between two points on a circle that pass through the center of the circle and is denoted d.
- **<u>Circumference</u>:** The circumference of a circle is the distance around the outside of a circle and is denoted C. Note that measuring circumference requires the use of π, an irrational number customarily approximated as 3.14.
- **<u>Volume</u>:** The volume of a three-dimensional object is a measure of the total space it occupies, measured in cubic units. Volume is represented by V.
- **<u>Surface Area</u>:** The surface area of a 3-D figure is the additive total of the area of all of the external surfaces of that figure.
- **<u>Sphere</u>:** A sphere is the 3-D equivalent of a circle that consists of all points in the three-dimensional space that are equidistant from a given center.

Key 2-D Formulas

- **<u>Perimeter</u>:** $P = s_1 + s_2 + \cdots + s_{n-1} + s_n$ for a polygon with n sides
- **<u>Area of a Parallelogram</u>:** $A = b \cdot h$
- **<u>Area of a Triangle</u>:** $A = \frac{1}{2}bh$
- **<u>Area of a Trapezoid</u>:** $A = \frac{1}{2}(b_1 + b_2)h$
- **<u>Pythagorean Theorem</u>:** $c^2 = a^2 + b^2$ where a, b, and c are sides of a right triangle
- **<u>Circumference of a Circle</u>:** $C = 2\pi r$ or $C = \pi d$ since $2r = d$
- **<u>Area of a Circle</u>:** $A = \pi r^2$

- **Example:** Find the area and perimeter of a Norman window (a rectangle joined with a semi-circle) who's base is 4 feet wide and rectangular height is 6 feet.
  Find the area of the base rectangle and add it to half of the area of a circle with a diameter the same as the rectangle's base.
  $$A_R = 4\text{ ft.} \cdot 6\text{ ft.} = 24\text{ ft.}^2$$
  $$A_C = \frac{1}{2}(3.14) \cdot (2\text{ ft.})^2 = 6.28\text{ ft.}^2$$
  $$A_R + A_C = 24\text{ ft.}^2 + 6.28\text{ ft.}^2 = 28.28\text{ ft.}^2$$
  Find the perimeter of the base rectangle, without the side adjacent to the semicircle, and add it to half of the circumference of a circle with a diameter the same as the rectangle's base.
  $$P = 4\text{ ft.} + 6\text{ ft.} + 6\text{ ft.} = 16\text{ ft.}$$
  $$C = \frac{1}{2} \cdot 2(3.14) \cdot 2 = 6.28\text{ ft.}$$
  $$P + C = 16\text{ ft.} + 6.28\text{ ft.} = 22.28\text{ ft.}$$

Key 3-D Formulas

- **<u>Volume of a Box/Cylinder</u>:** $V = A \cdot h$ where A is the area of the top or bottom of the container
- **<u>Volume of a Sphere</u>:** $V = \frac{4}{3}\pi r^3$
- **<u>Surface Area of a Sphere</u>:** $A = 4\pi r^2$
- **<u>Volume of a Cone/Pyramid</u>:** $V = \frac{1}{3}A \cdot h$ where A is the area of the base of the figure
- **Example:** Find the volume of a grain silo (a cylinder with a hemisphere on top) with a height of 40 feet and a diameter of 16 feet.
  Find the volume of the cylinder and add it to the volume of half of a sphere.
  $$V_C = Ah$$
  $$A = (3.14) \cdot (8\text{ ft.})^2 = 200.96\text{ ft.}^2$$
  $$V_C = 200.96\text{ ft.}^2 \cdot 40\text{ ft.} = 8{,}038.4\text{ ft.}^3$$
  $$V_S = \frac{1}{2} \cdot \frac{4}{3}(3.14) \cdot (8\text{ ft.})^3 = \frac{2}{3}(3.14) \cdot 512\text{ ft.}^3 = 1{,}071.787\text{ ft.}^3$$
  $$V_C + V_S = 8{,}038.4\text{ ft.}^3 + 1{,}071.787\text{ ft.}^3 = 9{,}110.187\text{ ft.}^3$$

Similar and Congruent Triangles

- ***Similar Triangles*:** Similar triangles have equal respective angles and proportional respective sides. If $\Delta ABC \sim \Delta DEF$, then $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$. Also, $\overline{AB}$ is proportional to $\overline{DE}$, $\overline{BC}$ is proportional to $\overline{EF}$, and $\overline{AC}$ is proportional to $\overline{DF}$. Similar triangles may be different sizes as long as the above criteria are met.

- ***Congruent Triangles***: Congruent triangles have equal respective angles and equal respective sides. If $\Delta ABC \cong \Delta DEF$, then $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$. Also, $\overline{AB} = \overline{DE}$, $\overline{BC} = \overline{EF}$, and $\overline{AC} = \overline{DF}$.
- **Note:** The order of the letters matter for both similar and congruent triangles. When comparing triangles in either case, the corresponding angles, represented by a capital letter, need to be lined up.

Trigonometry

- **<u>Trigonometric Ratios</u>:** Trigonometric ratios relate the length of the sides of a triangle to a particular angle, represented by θ. The hypotenuse is always the longest side. The opposite side is the side opposite to angle θ. The adjacent side is the side attached to angle θ.
  - **Sine:** $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
  - **Cosine:** $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
  - **Tangent:** $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
- **<u>Inverse Trigonometric Ratios</u>:** Inverse trigonometric ratios allow you to find the measure of θ when you know the length of the sides of the triangle.
  - **Arcsine:** $\theta = \sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right)$
  - **Arccosine:** $\theta = \cos^{-1}\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right)$
  - **Arctangent:** $\theta = \tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)$
- ***Calculator Tips***: If you have the angle measure given to you, use the appropriate trigonometric ratio. If you are asked for the angle measure, use the appropriate inverse trigonometric ratio. This usually requires pressing a "2nd" button, as the same button is used for both standard and inverse ratios. It is also important to ensure that your calculator is in degree (deg) mode as you make these calculations and not radian (rad) mode.
- **Example:** A telephone line runs from a telephone pole to the ground 30 feet away. The line forms a 33^{o} angle with the ground. How tall is the telephone pole?
  You are given the angle and adjacent side and are asked to find the opposite side. Use tangent as it has both things that are used in this problem.

$$\tan(33^{\text{o}}) = \frac{x}{30\text{ ft.}}$$

$$.6494 = \frac{x}{30\text{ ft.}}$$

$$x = 30\text{ ft.} \cdot .6494 = 19.482\text{ ft.}$$

Symbol Guide

Geometry Symbols		
Term	Symbol	Use
Area	A	$A = bh$
Base	b	See above
Height	h	See above
Side	a, b, c	$c^2 = a^2 + b^2$
Circumference	C	$C = 2\pi r$
Pi	π	Irrational number rounded to 3.14
Radius	r	See above
Diameter	d	$d = 2r$
Volume	V	$V = A \cdot h$
Angle Measure	$\angle$	$\angle A$
Triangle	Δ	$\Delta ABC \cong \Delta DEF$
Congruent	$\cong$	See above
Similar	$\sim$	$\Delta ABC \sim \Delta DEF$
Line Segment	$\overline{AB}$	
Theta	θ	Represents angle degrees